OAR:
The Basic Research Agency
of The United States Air Force

RESEARCH REVIEW
OFFICE of AEROSPACE RESEARCH

AF OFFICE OF SCIENTIFIC RESEARCH / A F CAMBRIDGE RESEARCH LABORATORIES / AERONAUTICAL RESEARCH LABORATORIES / EUROPEAN OFFICE O A R

NO. 15 8 OCTOBER 1962

Quantization of the Yang-Mills Field

General Physics Laboratory (ARL)

Many of the modern approaches to studying problems of theoretical physics are based on quantized field theories. Typical of this class is quantum electrodynamics which has enjoyed remarkable success in agreeing with and predicting experimental results in many areas involving electron interactions and electromagnetic radiation. There are still various undesirable features such as infinite divergences which must be manipulated out in obtaining solutions. In other cases, related to elementary particle physics and gravitational fields for example, the strongly non-linear field equations and the lack of full understanding of the force laws lead to serious problems in making further progress with such field theories. The importance of deeper understanding of these theories cannot be overemphasized, since the problems studied by means of them are at the forefront of physics today.

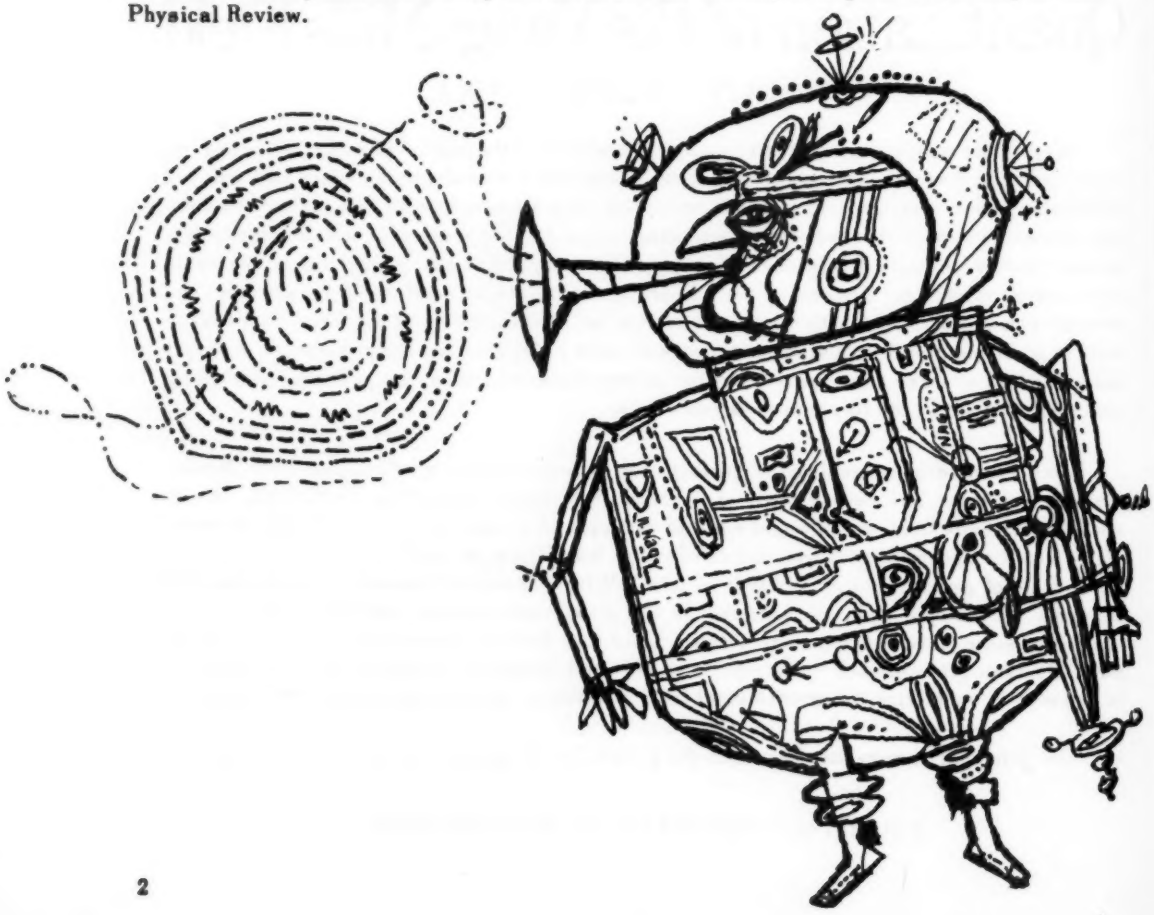
In order to attack the problems of quantization of the non-linear gravitational field theory, one approach taken has been a study of the somewhat simpler Yang-Mills (Y-M) field. This is a non-linear vector field, which would apply to particles of integral spin (bosons), like photons, pions, and the assumed gravitons, and is of gauge invariant type, again similar to the gravitational field of general relativity. Study of this Y-M field is also of interest in connection with problems of isotopic spin encountered in the strong interactions exemplified by nucleon-nucleon or some meson-nucleon interactions. Since this field is somewhat less complex mathematically than gravitational field, results on it permit acting as a guide to proper approaches to be taken in quantizing the gravitational field equations, and other non-linear field theories.

The principal inquiry has been into the possibility of quantizing the Y-M field, starting

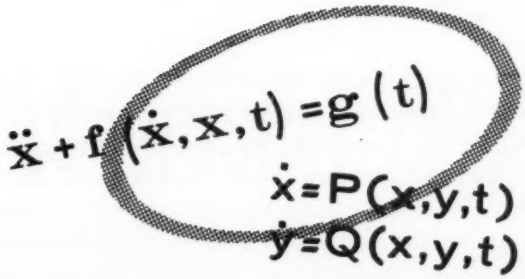
with the Dirac Lagrangian and necessary auxiliary conditions, and then securing the canonical form of the equations. To determine whether the quantization could be consistent it was then necessary to show the relations between the dynamical equations of motion and the constraint equations, which relations must be satisfied by commutation relations. In this case as a double commutator arises which is connected with the non-linearity of the theory. However, it was found that consistent quantization could be obtained for the particular conditions studied.

Further efforts have shown that, again for the particular cases studied, the theory is Lorentz invariant, a necessary condition for a physically meaningful relativistic theory. Also it has been shown that a coupling of this Yang-Mills field to the nucleon field could be performed which would satisfy the criteria for consistent quantization. Current efforts are concerned with studying this field in a different gauge to ascertain the changes which will take place in the commutator relations, and to see if the theory is equivalent under the two different gauge conditions. For the free electromagnetic field, this equivalence has been shown, but for the non-linear fields now under consideration the answer is not unambiguously known.

As a result of this work, one now has a better understanding of the problems encountered and required additional studies on the invariance properties of non-linear field theories. Such understanding is needed for further study of various proposed unified field theories which have often been hoped to lead to a more complete knowledge of the problems of elementary particles and gravitation. Details of portions of this study may be found in a paper by Professor R. Arnowitt of Syracuse University and Dr. S. Fickler of ARL, to be published shortly in the Physical Review.



MAKING FIXED POINTS SOLVE NONLINEAR DIFFERENTIAL EQUATIONS



DR. R. G. POHRER, Chief, Mathematics Division (AFOSR)

Everyone knows that topology can be used to solve differential equations. Not everyone has solved a differential equation by topological methods.

This dichotomy has a counterpart in two differing approaches to mathematics, the so-called abstract and the so-called applied. The fact that these two different approaches can be so delineated is in part due to a gap that exists between the creation of new mathematics and its reduction to the user level.

Bridging this gap is not a question, simply, of reading or writing a few choice words of explanation, or of finding or conveying an insight that unlocks a magic door between these two approaches. Bridging this gap is a question of research at an intermediate level concerned with developing the new mathematics in forms that may be applied to physical problems, in distinction to developing new mathematics in the abbreviated, simple and logically concise forms of the professional mathematician.

Bridging this gap between topology as a mathematical discipline and the solution of nonlinear differential equations, both ordinary and partial, has been accomplished by Professor Jane Cronin of the Polytechnic Institute of Brooklyn, under a Mathematics Division Contract awarded for this specific purpose. The result is a comprehensive development and exposition of topological fixed point theory and the theory of topological degree for the solution of differential equations. The manuscript has been accepted for publication in the "Survey Series" of the American Mathematical Society, and will appear under hard cover in the not too distant future.

In an illuminating introduction, Professor Scanlon examines the question of why topological methods are used in the solution of differential equations, the type of results they give, and when their use is appropriate.

The text develops the two major techniques of topological fixed

point theory and of local topological degree, and how to apply them. The four main subdivisions concern topological techniques in Euclidean n-space, the applications of this development to the solution of ordinary differential equations, topological techniques in function spaces, and the application of this development to the solution of integral equations, ordinary differential equations and partial differential equations.

There is no question but that this work is an important contemporary accomplishment. Its appearance in the AMS Survey Series undoubtedly assures it the widest possible dissemination and use.

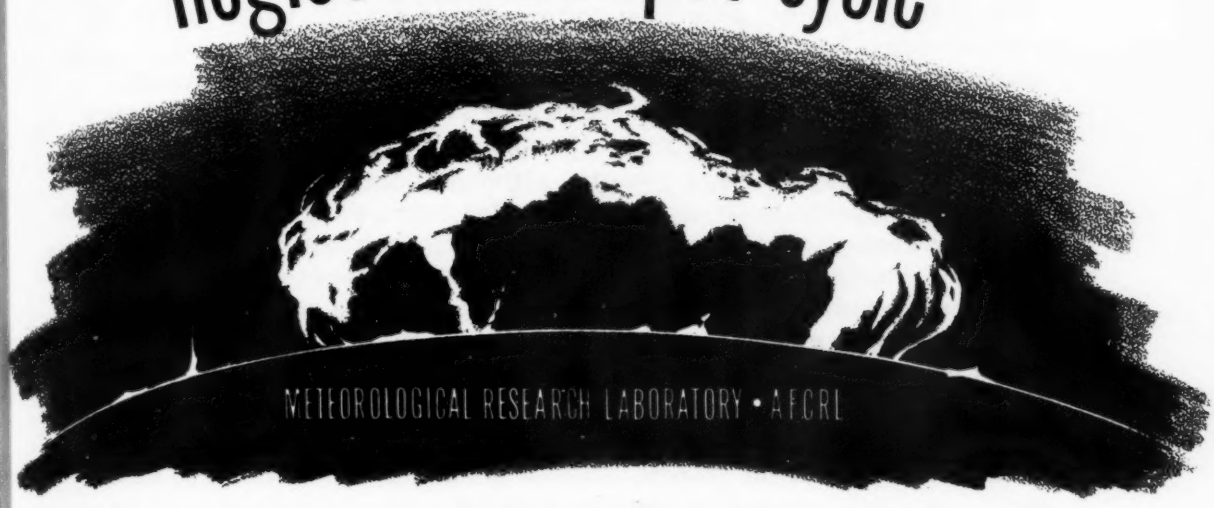
PRIZE PAPER OF THE JOINT AUTOMATIC CONTROL CONFERENCE

CAPTAIN B. S. MORGAN, JR., General Physics Division (AFOSR)

The American Automatic Control Council (AACC) recently voted a paper by Professor Charles A. Desoer and Jack Wing of the University of California, Berkeley, as the outstanding paper of the 1961 Joint Automatic Control Conference (JACC). The award, and the monetary prize which accompanies it, was presented to the authors at the 1962 JACC meeting in New York during the last week of June 1962. The prize-winning paper, entitled "A Minimal Time Discrete System," investigates the minimal-time regulator problem for a saturating sampled-data control system which has a linear plant with real and distinct characteristic roots. The investigators present a procedure for effecting the optimal control of such a system and describe an analog computer capable of generating this control. The research leading to the paper was jointly supported by the Electronics Research Directorate of AFCRL and the General Physics Division of AFOSR.

Conceived to reduce the overlap of conferences on control sponsored by individual societies, the annual JACC provides a unique high level conference on the theory and application of automatic control. The Sponsoring societies are the AIEE, IRE, ASME, ISA, and the AIChE.

neglected sunspot cycle



A recent observation causing considerable excitement among geophysicists is an apparent 26-month cycle in the direction of zonal winds in the equatorial stratosphere. Winds in this region seem to alternate over this period from a general easterly to a general westerly direction. The data collected on these oscillations, however, is only a few cycles in length -- covering about eight years. If future data exhibits essentially the same periodic behavior, an explanation must be found.

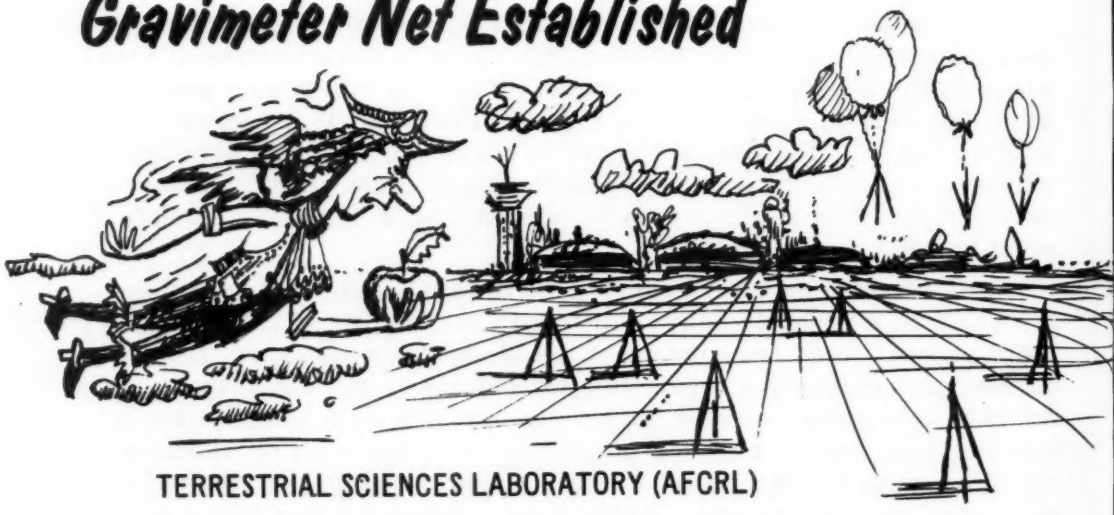
Ralph Shapiro and Frederick Ward of AFCRL have recently investigated a possible cause. Although the results of their study lend scant support to an explanation of the 26-month wind cycle, they uncovered a previously unsuspected periodicity in sunspot activity. The AFCRL scientists investigated sunspot activity as a possible explanation simply because the 26-month period of the wind oscillations would seem to argue against an explanation based on the internal dynamics of the atmosphere. Atmospheric dynamics would be a more likely cause were the observed cycles to occur over a 24-month period or bi-annually.

Variations of solar, ultraviolet radiation of the type associated with active sunspot groups could be expected to have their greatest effect near the equator at high levels. A long and consistant record of sunspot numbers over a 200-year period has been kept, and this record provided the basis for the Shapiro and Ward study. In the analysis of these sunspot records, they did in fact find some evidence

of periodicity in sunspot cycles in addition to the normal 11-year cycle. This cycle covers a 25-month period. The statistical results were shown to be significant at the five percent level. (It should be noted that certain a priori selection criteria were used to judge just what constituted a "peak" of sunspot activity.) The historical sunspot data was also analyzed in 50 and 100 year segments, and the 25-month cycle also appears in each of these sub-segments. The consistency of this cycle is somewhat more than might be expected by chance.

Unfortunately these results, although pointing up a previously unsuspected periodicity in sunspot cycles, throw little light on the 26-month period of oscillations in the zonal winds in the equatorial stratosphere. Perhaps it is just as well. Had the sunspot cycles coincided with the 26-month wind cycle, someone would then have to explain why the much more pronounced 11-year cycle of sunspot activity does not exert a corresponding influence on equatorial winds.

Gravimeter Net Established



TERRESTRIAL SCIENCES LABORATORY (AFCRL)

Gravity is the best understood and the least understood of physical phenomena. We know enough about it to satisfy the mathematics of celestial mechanics, but we know little of the mechanism of gravity itself. Nor do we know in refined detail gravity values over many parts of the continental U.S. In some cases, accepted values are in error.

Gravity values, of course, change with altitude and latitude. To

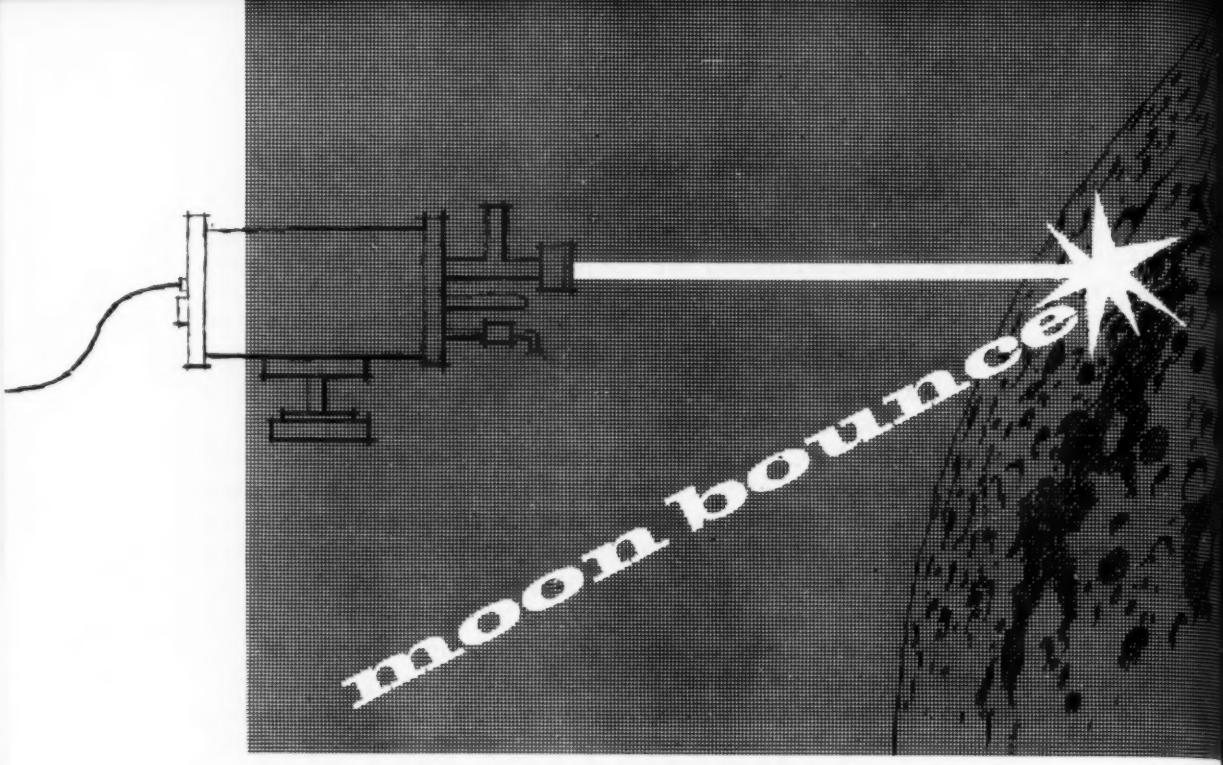
gain precise knowledge of gravity values, the approach is rather straightforward. Exceedingly accurate meters are designed and built and measurements are made at various geographical locations. Once base values are established for these locations, gravity surveys for surrounding areas are simplified.

Recently completed by AFCRL is a network of some 77 gravity base measuring stations. These stations will provide other military and civilian groups conducting gravimetric surveys with accurate base references. The AFCRL network, consisting of a series of triangular legs, are located at various Air Force bases and commercial airports. Work on this network has been in progress for several years under the direction of Dr. Lloyd G. D. Thompson, Major Charles S. Hawkins and Robert M. Perry of AFCRL.

Values for each station are arrived at by making a series of ground measurements. An aircraft has been periodically assigned to this project to transport the AFCRL gravity measuring instruments used in establishing this extensive network of gravity stations. Once the value for a particular station is established at some appropriate location on the air field, a pointed marker is installed. Values for these positions are published and are available at AFCRL. The base value for the AFCRL net is that of the Department of Commerce located at 14th Street in Washington, D. C., where g - 980.1182 miligals. The accurate base values established will assist future researchers in determining whether or not gravity values for a given location change with time -- a matter that has received some speculation.

The AFCRL network can be considered a part of a larger network encompassing gravimeter stations of the Coast and Geodetic Survey and stations established by Dr. Woolard of the University of Wisconsin. In addition to the AFCRL network of gravity measuring stations, AFCRL's work in gravimetry involves the design and development of increasing sensitive geodetic gravity meters, airborne gravity systems, absolute and relative pendulum equipment, accelerometers and gradiometers.

In September, a C-130 aircraft used by AFCRL in its airborne surveys was flown to Paris for static display at the "Meeting of the International Gravity Commission." At this meeting, AFCRL's work in the relatively new field of airborne gravity measurements was discussed.



MARSHALL C. HARRINGTON, General Physics Division (AFOSR)

Man's ability to reach into space has recently been advanced by the successful experiment of reflecting light from the moon. In May, scientists from the Research Laboratory for Electronics at the Massachusetts Institute of Technology (MIT) set up an optical maser at the Lincoln Laboratory of MIT. Bursts of red light were projected toward a dark portion of the moon in the second-quarter phase. The illuminated spot was so faint that it could not be seen by the eye, but the weak reflected light was detected and measured by the use of a telescope and a photoelectric cell.

The experimental team was headed by Professor Louis Smullin of MIT, a staff member of the Research Laboratory for Electronics, and Dr. Georgio Fiocco. The participation of the laboratory in the experiment was supported in part by the Air Force Office of Scientific Research.

The development of the optical maser came as an extension into the range of visible light of earlier work in stimulated emission in the microwave portion of the electromagnetic spectrum. The word "maser" is an acronym for "molecular amplification by stimulated emission," an effect which was investigated under sponsorship of the Air Force Office of Scientific Research by Dr. Charles H. Townes, former Pro-

fessor of Physics at Columbia University, and now Provost at MIT.

The reflection of radio waves from the moon had been accomplished some years ago, but until recently the lack of a powerful coherent source made long range light beams impractical. The development of the optical maser, or "laser," as it is often called, about two years ago provided the possibility of creating a nearly parallel beam of coherent light which could travel long distances without spreading as do ordinary beams. The experiment was designed to determine whether available devices were sufficiently powerful to reach several hundred thousand miles into space. The moon was selected as a convenient target.

The optical maser used in this demonstration contained a ruby crystal, which received light from high-intensity flash lamps, stored the energy very briefly in elevated atomic states, and then emitted a burst of "coherent" red light of high intensity. Each burst produced a 50-joule beam, the most powerful yet reported for an apparatus in a portable form in this relatively new field of optical maser technology. The light was polarized and had a single wave length. The ruby crystal, a rod about six inches long and 3/8 of one inch in diameter, was surrounded by an array of four xenon gas tube lamps, each with a reflector to focus the light on the ruby. The ends of the rod were polished flat and one end was coated so that the light was emitted at the opposite end. Thirteen bursts of light, each lasting about 1/2000th second, made the round trip to the moon, a total distance of approximately 500,000 miles, in 2 1/2 seconds. Some of the energy of the light beam was scattered or absorbed by the earth's atmosphere on the way, and some was lost at reflection at the moon's surface. It is estimated that of the some 200 billion trillion photons of light in each burst, only about twelve returned to the receiving telescope to be focused, and amplified.

The optical maser holds great promise for the future. The Air Force is interested in many of its present and proposed developments. Among these are high performance radar, communication and surveillance systems, data processing, applied metrology, selective and precise welding of materials, and applications in surgery, especially in the eye. Still more distant developments can be imagined, such as the use of optical maser beams for navigation and guidance of space vehicles and weapons employing high intensity beams for destroying missiles and satellites. Major technological problems would have to be solved to achieve these last results, but in the meantime, applications now within reach justify the investment of time and effort which is being placed in exploiting this field of research.

BATTLESHIP, ANYONE ?

COMPUTER AND MATHEMATICAL SCIENCES LABORATORY (AFCRL)

Games -- chess, checkers, bridge and so on -- provide perhaps the best means for evaluating the performance of a computer in skills requiring adaptability and intelligence. The effectiveness of computer judgment in selecting the best course of action from a set of alternatives can be directly weighed against the performance of the computer's human opponent.

A game particularly well-suited for evaluating the attributes of adaptability and intelligence is Battleship. H. M. Willett and S. R. Petrick of AFCRL have recently reported the results of a series of battleship engagements between the AFCRL Cambridge Computer and various human opponents.

There are two aspects of Battleship: hiding one's own ships and searching for enemy ships. The strategy should make use of relevant probabilities in searching for enemy ships, but it should also exhibit adaptive behavior to permit the use of observed similarities in an opponent's placement of ships. (It should be explained that

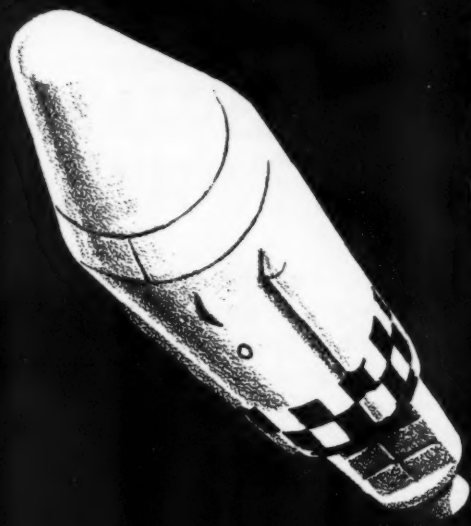
Battleship is played on a 10 by 10 grid of 100 squares. Four ships of three, four, five and six squares are placed on the grid by the participant in such a way that, it is hoped, the opponent will have difficulty in finding them.

If we assume a random distribution of ships, we may compute the probability that a ship of largest possible length is incident to a particular square. This is done by dividing the total number of ships which are of length equal to the number of squares in the largest unsunk ship and which include that square, by the total number of ships of that length which can be placed on the board at the time. If we are given a posteriori probabilities that a ship is oriented in the vertical, horizontal or diagonal directions, the preceding probability is easily modified to make use of this information through the use of conditional probabilities.

A number of studies have been conducted to determine the effectiveness of the various programs relative to that of human competitors. In one sequence of games, in which targets were placed on the board in a random fashion, the computer averaged 37.56 shots per game as compared to 40.08 for the eleven subjects.

In sequence of twelve games with similar biases in ship placement common to all of the games, the subjects discovered and made use of the similarities which were neither planned nor recognizable by some of the computer programs. They averaged 40.05 shots as opposed to 35.21 shots for the computer. The computer made distinct improvement as more games were played. For example, the average dropped from 37.50 for the first games to 32.92 for the last six. The latter figure approached a "perfect learning" average of 31.78 shots. This figure was obtained by supplying the computer with true a priori probabilities for each game rather than using approximating a posteriori probabilities obtained from past games.

It is hoped that this study will be of interest to persons engaged in extending digital computation to other areas requiring heuristic searching or adaptive behavior.



research in celestial mechanics

CAPTAIN B. S. MORGAN, JR.
Applied Mathematics Division
(AFOSR)

The ability to create artificial celestial bodies has led to a recent revival of interest in celestial mechanics. Because the characteristics of the orbits of these bodies often differ very radically from those of the natural bodies, new problems have been suggested as well as a revival of interest in older problems. In some cases, while the motion of an artificial body resembles that of a previously studied natural one, the scale of the motion is so different as to render the old solution unsuitable. For example, many of the classical expansions converge for intervals of time of the order of hundreds or even thousands of years. However, artificial satellites have periods measured in hours rather than in months or years, so that for such a satellite the convergence of similar solutions may be assured only for months or less. That is, the classical methods thought to be valid for centuries for planetary motion are probably valid for weeks for artificial satellites. Since artificial satellites are in orbit for years and we desire to determine their orbits mathematically with high accuracy, it is important to seek solutions which are valid for an extremely long time, or better, for all time.

One fairly straightforward method yields results which are accurate (up to terms of the second order) for all time. This method, by Krylov and Bogoliuboff, has as its key the replacement of a slowly varying derivative by its average over one cycle of the independent variable. In the averaging process, the dependent variables are considered constant. As a result, the so-called

first approximation is obtained. This approximation has generally been found to be adequate for physical problems, (the comment above on accuracy applies to the "improved first approximation" or second order terms).

As part of the AFOSR Applied Mathematics Division's support of research in celestial mechanics, Mr. George Shapiro of Westinghouse Electric Corporation is investigating analytical techniques, including the Krylov-Bogoliuboff method, that are applicable to certain problems in celestial mechanics. Problems of interest are the determination of the relative motion of two satellites of the Earth or of an Earth-satellite and another heavenly body as well as the capture, collision and stability of orbits in Earth-Moon space.

So far, closed form solutions to the equations of the first approximation of orbits with low tangential thrust have been obtained by Mr. Shapiro (See AFOSR Documents 1479, 1823, and 2610) using the method of Krylov and Bogoliuboff. (A solar observation satellite under the pressure of solar radiation is an example of a satellite with an orbit with a low tangential thrust.)

Five Nike Cajun sounding rockets carried AFCRL experiments aloft in August. One of these was fired from the Eglin Gulf Test Range during the week of 27 August. The rockets carried instruments for the investigation of absorption of 1 Mcps signals by the D-region of the ionosphere. Each carried a 55-pound payload to an altitude of 90 miles. (Propagation Sciences Laboratory). . . .In a program conducted jointly with NASA and Swedish research groups, four Nike Cajun rockets were launched in Sweden during August. Payloads were carried to 70 miles to investigate noctilucent clouds. AFCRL designed most of the instrumentation and furnished the instrument packages for the experiment. (Space Physics Laboratory)

Recently published by Stanford University, under AFCRL contract, is a detailed bibliography of the ionosphere, a survey of all the scientific literature on ionospheric research. The bibliography had been in process of preparation for several years. (Propagation Sciences Laboratory)

RECORD-BREAKING BALLOON FLIGHTS

RESEARCH INSTRUMENTATION LABORATORY (AFCRL)

An AFCRL balloon, launched from Bermuda on May 31, set new records for flight duration and distance. The balloon remained aloft for 19 days, terminating its flight on 19 June, near Iwo Jima, 9300 miles from its Bermuda launch site.

The balloon is of a new type and represents a major advance in balloon technology. Key factor in the record-breaking flight is the balloon's ability to maintain a constant altitude over periods of weeks without the use of ballast. The balloon maintained a constant altitude of 68,000 feet during the 19-day flight with altitude variations of no more than plus or minus 200 feet. Balloons of this new type are not affected, as are conventional balloons by variations in the day-night temperature. These variations, which require the use of ballast to compensate for the loss of altitude at night, set a limit on the time that a conventional balloon can remain aloft.

To maintain a constant altitude, the balloon is superpressurized. The pressure inside the balloon exceeds that of the outside pressure. This means that the balloon structure must be capable of containing the lifting gas under positive varying pressure without any change in volume as the balloon warms and cools during flight. Flight duration is limited only by the eventual deterioration of the balloon material.

The material used in the 34-foot balloon is Mylar. The 50-pound payload consisted of performance-monitoring instruments to measure temperatures and pressures of lifting gas, a data-encoder and transmitter, and a radio-command receiver which receiver which received the cut-down signal. All of the automatic equipment functioned perfectly during the long flight. Performance data indicated that the balloon could have continued aloft for a much longer period of time.

The balloon was kept under constant radio surveillance by groups from the Air Force Cambridge Research Laboratories located at Holloman Air Force Base, New Mexico, and at Chico, California, from which the cut-down signal was transmitted. Tracking

and data recording were performed by a network of stations of the Federal Communications Commission.

On 10 June, AFCRL launched a second superpressure balloon from Bermuda. This second balloon was of the same design as the first and carried the same type of equipment. It is expected that the flight duration of this second balloon will exceed that of the first.

Balloons of this type will provide an ideal test bed for many kinds of research projects related to meteorology and to studies of atmospheric characteristics at high altitudes. An additional and obvious advantage of these vehicles is their low cost.

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